AN EQUIVALENCE OF DG-DERIVED DEFORMATIONS

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Abstract

In ongoing joint work with Wendy Lowen, Michel Van den Bergh and Francesco Genovese, we aim to further develop and understand the deformation theory of pretriangulated dg-categories with a sufficiently nice t-structure as introduced in [GLV21; GLV22]. Our main theorems establish an equivalence of deformation pseudofunctors, relating so-called t-deformations of such a (bounded) pretriangulated t-dg-category $\mathcal{A}$ to classic dg-deformations of the full dg-subcategory of derived injectives DGInj(h-proj($\mathcal{A}$)) of h-proj($\mathcal{A}$) (these can be regarded as the building blocks, much like the injective objects in the abelian setup). Pointwise, this amounts to an equivalence

$$\mathrm{Def}^{\mathrm{t}}_{\mathcal{A}}(\theta) \cong \mathrm{Def}^{\mathrm{dg}}_{\text{DG-Inj(h-proj}(\mathcal{A}))}(\theta), \tag{1}$$

where $\theta : R \to S$ is a base change morphism of dg-rings. This would then be a dg-derived analogue of the equivalence that was established in [LV06] in the abelian setting.

I will commence the talk by discussing our motivation for (1), which stems from the fact that it provides a deformation-theoretic interpretation[1] of the higher Hochschild Cohomology groups $\mathrm{HH}^n(\mathbb{A})$, $n \geq 3$, of an abelian category $\mathbb{A}$. Next, I will outline our approach, drawing parallels with the abelian story. Since one direction of the equivalence (1) has been addressed in [GLV22] using the reconstruction theorems of [GLV21] – namely that a dg-deformation of the dg-category of derived injectives induces a t-deformation between the pretriangulated t-dg-categories – I will focus on the converse. The aim is to provide an overview of the proof and its various components and mention the technical issues we are still facing. I will treat one key component in more detail: a dg-enhancement of the derived category $\mathcal{D}(\mathcal{A})$ of a pretriangulated dg-category in terms of filtered homotopy colimits. This enhancement allows us to extend t-structures from $\mathcal{A}$ to $\mathcal{D}(\mathcal{A})$.

References

[GLV21] F. Genovese, W. Lowen, and M. Van den Bergh, "T-structures and twisted complexes on derived injectives," *Adv. Math.*, vol. 387, Paper No. 107826, 70, 2021.

[GLV22] F. Genovese, W. Lowen, and M. Van den Bergh, *T-structures on dg-categories and derived deformations*, 2022. arXiv: `2212.12564 [math.CT]`.

[LV06] W. Lowen and M. Van den Bergh, "Deformation theory of abelian categories," *Trans. Amer. Math. Soc.*, vol. 358, no. 12, pp. 5441–5483, 2006.

[1] Recall that there is an interpretation of $\mathrm{HH}^{2,3}(\mathbb{A})$ in terms of flat infinitesimal abelian deformations and obstructions thereof when extending to formal ones.